Heuristic approach to the airline schedule disturbances problem: multi-fleet case

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Abstract In order to reduce negative effects of flight schedule disturbance, such as meteorological conditions, aircraft failure, etc., a decision support system (DSS) – multi-fleet case (ASDP-MFC) is developed aiming to assist the decision makers in handling disturbances in real-time. This system is based on a heuristic algorithm which generates a list of different feasible schedules ordered according to the value of an objective function. The possibilities of DSS are illustrated by real numerical examples that concern middle size European airline’s flight schedule disturbances.

1 Introduction

The daily schedule is the final version of the flight schedule for the considered day, which takes into account all the changes that were undertaken during the period from its publication till just before its realization. As the flight schedule is realised, disturbances occur that cannot be predicted by the airline in advance. When disturbances make it impossible to realize the planned flight schedule, the dispatcher at the airline operational centre (AOC) defines a new daily operational flight schedule. The airline schedule disturbance problem – multi-fleet case (ASDP-MFC), which is faced by dispatchers at the AOC day-to-day, is considered in this paper. This research presents extended work on the Airline Schedule Optimization project [2] in which the basic ASDP model is developed. During the ASDP-MFC solving the dispatcher at the AOC considers airline policy and defines a new flight schedule that will optimize an objective function and satisfy the corresponding constraints. Depending on airline policy, when designing the new flight schedule, the dispatcher delays or cancels some flights and reassigns flights to available aircraft in order to reduce the negative effects of disturbance.

The airline flight schedule designing in the case of disturbances is a problem that has been addressed in literature with several different models, objective functions, constraints and assumptions, as well as different solution approaches. Stojković [11] and Teodorović and Stojković [13] developed several models of airline schedule recovery problems and the corresponding exact methods and heuristics for solving them. Stojković et al. [12] developed a model which can be solved to optimality in a real-time, but only in cases of small disturbances. Rakshit et al. [9] presented a decision support system incorporating the models considered by...

This paper presents a mathematical formalization for the airline schedule disturbances problem and a developed heuristic algorithm for generating new daily operational flight schedules. The corresponding DSS is developed aiming to assist the dispatcher in handling disturbances in a real time. The mathematical model is defined in such a way that by changing the objective function, it can support different decision-making strategies of the airline. The proposed heuristic algorithm offers a list of different feasible solutions ordered according to the value of an objective function. During the decision-making process the dispatcher could define a new strategy, resolve a problem and select a solution. The paper has five sections: the introduction and a literature review are followed by the definition of the problem and its mathematical formalization in Section 2. A heuristic algorithm proposed for its solution is described in Section 3. In the fourth and fifth sections a numerical example and conclusions, including further research recommendations are presented.

2 Problem definition and mathematical formalization

In the case when a disturbance has occurred at an airport or on an aircraft and consequently some flights can no longer be realized according to the planned daily schedule, a new departure time shall be defined and/or a new aircraft shall be assigned to each such a flight, or it shall be cancelled, in such a way that the objective function is minimized. This function represents the additional costs of the airline caused by disturbances. In this paper the additional costs consist of six elements: the aircraft swapping cost, the priority and the non-priority flight cancellation cost, the passengers delay cost, the aircraft regular maintenance disturbance cost, and the aircraft balance cost. In order to overcome disturbances and find a new daily operational flight schedule, some assumptions are introduced:

- A disturbance may occur at an airport or on an aircraft. All disturbed airports and aircraft are identified and the corresponding starting time for a new flight schedule is defined.
- The flight schedule recovery period, for which a new flight schedule is to be designed, is the time period from the starting time, corresponding to the identified disturbance, to the last moment at which the timetable for the next day is not disturbed.
- A rotation is a sequence of flights where the first flight in the series departs from the base airport, and the last one arrives at the base airport.
- The airline has a fleet which consists of different types of aircraft characterized by different seat capacities, where aircraft of the same type have the same seat capacity; all aircraft can be divided in three categories: aircraft for short-haul (cat. 1), aircraft for mid-haul (cat. 2) and aircraft for long-haul flights (cat. 3). All three categories consist of different aircraft types; the aircraft ground handling time depends on the aircraft type and the airport where the handling occurs.
- Aircraft can be swapped – bigger aircraft can service flights originally assigned to smaller ones (seat capacity is not smaller) and if is from the same or the first following category in relation to originally assigned aircraft.
- There are no spare aircraft in the fleet; a set of priority flights is given (flights with the slot time, transfer passengers, etc.); the maximal allowed delay is defined for each flight; ferry flights (flights without passengers) are not allowed; crew constraints are not considered.
• Priority flights can be realized according to the so-called VIA principle. If flights \( i \) and \( j \) are planned flight, where \( j \) is a priority flight from airport \( B \) to airport \( C \), while \( i \) is a non-priority one from airport \( A \) to airport \( C \) and if at airport \( B \) the aircraft assigned to flight \( i \) has a failure, then the aircraft assigned to flight \( j \) can realize an additional unplanned flight \( i' \) (from \( A \) to \( B \)), and then priority flight \( i \). In this way that aircraft can service both flight \( i \) and flight \( j \), but only if its seat capacity is not smaller than the total number of passengers on these two flights. Therefore, for each planned priority flight (flight \( i \)) a set of additional non planned flights (flights \( i' \)), which can by used for its realization applying the VIA principle, is given in advance. Obviously, such an additional flight should have the destination airport identical to the origin airport of the priority flight. Also, there should exist at least one non-priority flight (flight \( j \)) with the origin and the destination airports identical to that of the origin airport of the additional flight and the destination airport of the priority flight, respectively. The cost of passenger’s delay of priority flight (if delay exists) calculates as product of priority flight delay, sum of passengers on both flights and average passenger’s delay cost per time unit.

• For each flight the average passenger’s delay cost per time unit (equal for all flights and all passengers) and the cost of aircraft type swapping are known in advance. The cancellation costs are different for priority and non-priority flights.

Starting with the previous assumptions, the following constraints of the problem ASDP-MFC are specified: (1) *Time constraints*: Each flight in a new flight schedule should not depart earlier than the known departure time planned before the disturbance occurrence. If the departure time of a flight in a new schedule is delayed with respect to the planned time, this delay should not be greater than the maximal delay allowed for this flight. The time period between two consecutive flights in the rotation of an aircraft should be sufficient for its ground handling. The take-off and landing of each flight in the new schedule should be serviced at the corresponding airport by the end of its working hours, i.e. by its closing time valid on the day for which this schedule is designed. (2) *Aircraft maintenance constraints*: Each aircraft, planned to have the regular technical maintenance at the end of the flight schedule recovery period, should finish its rotation at the defined airport where the maintenance is performed. (3) *Aircraft balance constraints*: In order to successfully service the planned flights after the recovery period, it is necessary that at the end of this period certain types of aircraft are available at each airport in sufficient numbers (Otherwise, the disturbance will be extended to the following day). This condition can be satisfied by requiring that, for each of such types, a given number of aircraft with the seat capacity not smaller than the seat capacity characterizing this type, should finish their rotation at the given airport. (4) *Capacity constraints*: The number of passengers on each flight should not be greater than the seat capacity of the aircraft assigned to this flight. The seat capacity of an aircraft, used to service a priority flight applying the VIA principle, should not be smaller than the total number of passengers on that flight and a non-priority flight that is indirectly realized in this way.

In order to make the problem definition more precise and easier to understand, we will give a mathematical formalization of the objective function and the main constraints of the problem ASDP-MFC.

Let us introduce the following notations of its input parameters: \( F \): a set of flights which should be serviced in the recovery period; \( Pr \): set of priority flights, \( Pr \subset F \); \( VIA(i), i \in Pr \): set of additional flights which can be used in the realization of priority flight \( i \) using the VIA principle, \( VIA(i) \cap F = \emptyset \); \( id(i,i'), i \in Pr, i' \in VIA(i) \): a non-priority flight which can be indirectly realized by servicing flights \( i' \) and \( i \) using the VIA principle, \( id(i,i') \in F' \); \( F' = F \cup \bigcup_{i \in Pr} VIA(i) \); \( AP \): set of airports; \( o(i), d(i), i \in F' \): the origin and the destination airport of flight \( i \), \( o(i), d(i) \in AP \); \( TP(i), i \in F \): the departure time of flight \( i \) planned before the disturbance occurrence; \( delay(i), i \in F \): the maximal allowed delay of flight \( i \); \( pax(i), i \in F \): the number of passengers on flight \( i \); \( k(i), i \in Pr \): the cancellation cost of priority flight \( i \); \( k_2(i), i \in F \backslash Pr \):
The cancellation cost for non-priority flight \( i; k \): the delay cost of a flight per a time unit; \( k_p \): the cost for passenger’s delay per time unit; \( d(i,l) \): the cost of servicing flight \( i \) by aircraft of type \( l \); \( \{ \) \( TYPE \); \( j(i) \): the aircraft which should service flight \( i \) according to rotation plan; \( AC \): set of aircraft available for servicing flights from \( F' \); \( TYPE \): set of aircraft types; \( atype(j) \), \( j \in AC \): the type of aircraft \( j \), \( atype(j) \in TYPE \); \( cap(l) \), \( l \in TYPE \): the seat capacity of an aircraft of type \( l \); \( cat(l) \), \( l \in TYPE \): the category of an aircraft of type \( l \):

\[
cat(l) = \begin{cases} 
1 & \text{short - haul} \\
2 & \text{mid - haul} \\
3 & \text{long - haul}
\end{cases}
\]

\( a(l,k) \), \( l \in TYPE \), \( k \in AP \): the ground handling time for an aircraft of type \( l \) at airport \( k \); \( dis(j) \), \( j \in AC \): the airport where aircraft \( j \) is located at the starting time of the recovery period, \( dis(j) \in AP \); \( t(j) \), \( i \in F', j \in AC \): block time (the time between engine start at the airport of origin and engine stop at the airport of destination) or the duration of flight \( i \) serviced by aircraft \( j \); \( MNT \): set of aircraft which should finish its rotation at certain airports for the regular technical maintenance, \( MNT \subset AC \); \( mnt(j) \), \( j \in MNT \): the airport where aircraft \( j \) should finish its rotation, \( mnt(j) \in AP \); \( clot(k) \), \( k \in AP \): the closing time of airport \( k \) valid on the day for which a new schedule is designed; \( PTYPE(k) \), \( k \in AP \): set of aircraft types which should be available at airport \( k \) at the end of the recovery period; \( notype(l,k) \), \( l \in AP \), \( l \in PTYPE(k) \): the number of aircraft with the seat capacity not smaller than \( \text{cap}(l) \), which should finish their rotation at airport \( k \); \( pen(l,k) \), \( k \in AP \), \( l \in PTYPE(k) \): the penalty cost per aircraft when the number of aircraft, with the seat capacity not smaller than \( \text{cap}(l) \), which finish their rotation at airport \( k \), is smaller than \( \text{notype}(l,k) \); \( DAC \): set of airports with detected disturbances, \( DAC \subset AP \); \( TDAP(k) \), \( k \in DAC \): the earliest possible departure time of aircraft \( k \) from airport \( dis(j) \) after the disturbance elimination; \( DAC \): set of aircraft with detected disturbances, \( DAC \subset AC \); \( TDAC(j), j \in DAC \): the earliest possible departure time of aircraft \( j \) from airport \( dis(j) \) after the disturbance elimination;

The variables of the problem ASDP can be defined in the following formal manner.

- \( rot(l,j), rot(2,j), ..., rot(l(j),j) \): the rotation of aircraft \( j \), where \( l(j) \) is the number of flights in this rotation, \( rot(l(j)) \in F' \) for \( l=1, 2, 3, ..., l(j) \), and \( j \in AC \);
- \( TR(i), i \in F' \): the earliest possible departure time for flight \( i \);
- \( X(i,j), i \in F', j \in AC \): a binary variable equal to 1 if aircraft \( j \) is assigned to flight \( i \), 0 otherwise; \( can(i) \), \( i \in F' \): a binary variable equal to 1 if flight \( i \) is cancelled, 0 otherwise; \( mnt(i), i \in MNT \): a binary variable equal to 1 if aircraft \( j \), requiring the regular technical maintenance, has not finished its rotation at airport \( mnt(j) \), 0 otherwise; \( sat(s,l,j,k) \), \( k \in AP, j \in AC \), \( l \in PTYPE(k), s \in \{1, 2, ..., \text{notype}(l,k)\} \): a binary variable equal to 1 if aircraft \( j \), finishing its rotation at airport \( k \), satisfies the \( s \)-th necessity of this airport for aircraft type \( l \), 0 otherwise.

Using the previously introduced notations the objective function of ASDP-MFC can be formally expressed by relation (1):

\[
\min F = \sum_{a \in AP} \sum_{j \in AC} (d(i, atype(j)) - d(i, atype(j')))X(i, j) + \sum_{i \in F'} k(i) can(i) + \\
+ \sum_{i \in F'} k(i) can(i) + k_p \sum_{i \in F'} (TR(i) - TP(i)) pax(i)(1 - can(i)) + \\
+ \sum_{j \in MNT} k(j) mnt(j) + \sum_{j \in AC} \sum_{l \in PTYPE(k)} pen(l,k)(notype(l,k) - \sum_{s=1}^{\text{notype}(l,k)} sat(s,l,j,k))
\]

The first term in (1) represents the cost of aircraft type swapping, while the other terms are the priority-flight cancellation cost, the non-priority flight cancellation cost, the passenger’s delay cost, the aircraft maintenance disturbance cost and the balance disturbance cost, respectively. Let us notice that, defining the objective function in the form (1), we in fact relax the aircraft maintenance and the aircraft balance constraints, introducing the total
penalized violation of these constraints as a part of its cost. In this way, although the constraints can be unsatisfied, the maximization of the objective function tends to minimize their violation.

The main constraints of ASDP-MFC can be formally defined by (2)-(14).

\[ TR(i) \geq TP(i), \quad \text{for } i \in F, \quad (2) \]
\[ TR(i) - TP(i) \leq \text{delay}(i), \quad \text{for } i \in F, \quad (3) \]
\[ TR(\text{rot}(l,j)) + t(\text{rot}(l,j),j) + a(\text{atype}(j),d(\text{rot}(l,j))) \leq TR(\text{rot}(l+1,j)), \quad \text{for } l=1, 2, \ldots, l(j)-1, j \in AC, (4) \]
\[ d(\text{rot}(l,j)) = o(\text{rot}(l+1,j)), \quad \text{for } l=1, 2, \ldots, l(j)-1, j \in AC, \]
\[ o(\text{rot}(l,j)) = d(\text{rot}(l,j)), \quad \text{for } j \in AC, \]
\[ TR(i) \leq \text{clo}(o(i)), \quad \text{for } i \in F'. \]
\[ TR(i) + t(i,j) \leq \text{clo}(d(i)), \quad \text{for } i \in F', j \in AC, X(i,j)=1. \]
\[ TR(i) \geq \text{TDAC}(j), \quad \text{for } j \in AC, X(i,j)=1; \]
\[ TR(i) \geq \text{TDAP}(o(i)), \quad \text{for } o(i) \in \text{DAP}, \]
\[ TR(i) \geq \text{TDAP}(d(i)) - t(i,j), \text{for } d(i) \in \text{DAP}, X(i,j)=1, \]

The conditions (2)-(11) express the time constraints of the problem: (2) and (3) mean that each flight departs not earlier than the planned departure time, while its delay is not greater than the maximal allowed value. Constraints (4) indicate that the following flight in the rotation of an aircraft cannot take off before the previous flight has landed and the aircraft has been ground-handled. Equalities (5) and (6) provide that in the rotation the destination airport of a flight and the origin airport of the following flight are identical, as well as the first flight starts from the airport where the aircraft is located stays at the starting time of the recovery period. Inequalities (7) and (8) express that a flight should take-off before the closing of its origin airport and it should not land after the closing of its destination airport. Also, a flight cannot take off before repairing a breakdown of the aircraft assigned to it (9). In the case when its origin airport has a detected disturbance, a flight cannot take off before the airport is reopened (10), while if its destination airport is disturbed, it cannot land before the disturbance has been eliminated (11).

\[ \text{cap}(\text{atype}(j)) - \text{cap}(l) \geq 0, \quad \text{for } \text{sat}(s,l,j,k)=1, \quad k \in AP, \quad j \in AC, \quad l \in \text{PType}(k), \quad s \in \{1, 2, \ldots, \text{notype}(l,k)\}. \quad (12) \]

Inequalities (12) are related to the aircraft balance constraints and provide that each aircraft, which satisfies the need for a certain aircraft type at an airport to have the seat capacity not smaller than the seat capacity characterizing this type.

\[ \text{cap}(\text{atype}(j)) \geq \text{pax}(i), \quad \text{for } i \in F, \quad j \in AC, \quad X(i,j)=1; \quad (13) \]
\[ \text{cap}(\text{atype}(j)) \geq \text{pax}(d(i,i)), + \text{pax}(i), \quad \text{for } i \in \text{Pr}, \quad i \in \text{VIA}(i), \quad X(i,j)=1. \quad (14) \]

Conditions (13) and (14) are the capacity constraints: (13) means that the number of passengers on each flight is not greater than the seat capacity of the assigned aircraft, while (14) expresses that the total number of passengers on that flight and an indirectly realized non-priority flight is not greater than the seat capacity of the assigned aircraft, when a priority flight is realized using the VIA principle.

\[ \text{cap}(\text{atype}(j)) \geq \text{cap}(\text{atype}(j^*(i))), \quad \text{for } i \in F, \quad j \in AC, \quad X(i,j)=1; \quad (15) \]
\[ 0 \leq \text{cap}(\text{atype}(j)) - \text{cap}(\text{atype}(j^*(i))) \leq 1, \quad \text{for } j \in AC, \quad i \in F, \quad X(i,j)=1. \quad (16) \]

The flights assigned to an aircraft, in the case of disturbance, can be performed only by aircraft which capacity is not smaller than the capacity of originally aircraft (15), and if that new aircraft is from the same or from the first following category (16).

The mathematical modelling of ASDP-MFC, defined by (1)-(16) is only partial, hence many constraints, which are assumed to be satisfied, are not formalized. For example, the objective function (1) is correct only if we assume that an aircraft cannot be assigned to a cancelled flight, i.e. \( \text{can}(i)=1 \), if and only if \( X(i,j)=0 \) for each \( j \in AC \). But, this mathematical formalization could be a basis for developing more sophisticated mathematical models of the ASDP problem, such as a mixed integer programming or a constraint programming model. As this problem is known to be NP-hard [5] and it should be solved in a real time, instead of making a great effort to find such an appropriate model and try to solve it using an exact method, we focus on a heuristic approach to the problem. Therefore, in the next section we
propose a special heuristic technique for determining a list of feasible “satisfactory” (sub-optimal) new daily flight schedules, among which the dispatcher can select and implement the most convenient one.

3 Heuristic algorithm for ASDP-MFC

Before presenting the steps of a proposed heuristic algorithm for ASDP-MFC, we will describe in more details some basic notions introduced in Section 2. In this paper rotation is defined as a one-day rotation or part of multi-day rotation during a considered day. Rotations consist of mini rotations and simple rotation segments. A mini rotation is a series of flights attached to each other where the departure airport of the first flight is the same as the arrival airport of the last flight in the series (A-B-A). A simple segment of the rotation is the series of flights attached to each other where the departure airport of the first flight and the arrival airport of the last flight in the series are different airports (A-B-C).

The results of disturbances are flight delay and/or cancellation. The flight can be cancelled temporarily or permanently. A temporarily cancelled flight can be realized by adding its cancelled mini rotation/simple segment to the rotation of some of the other aircraft (considering departure airport, arrival airport and time constraints) before the first flight in the rotation, between flights in the rotation, or after the last flight in the rotation. The temporarily cancelled flights and its cancelled mini rotations/simple segments are permanently cancelled if there is no possibility to add them to the rotation of one of the other aircraft by any of previously described ways. Also, a priority flight and its mini rotation or all following flights of its simple segment by the end of the considered day, are permanently cancelled if there is no possible way to realize it using the VIA principle.

In order to reduce the total delay and/or the balance disturbance cost, the delayed flights can be crossed. Crossing delayed flights can be achieved by two operations: removing part of one and adding it to the other rotation, and interchanging parts of two rotations. Removing part of one and adding to the other rotation refer to the delayed flights whose delay is not directly caused by the airport disturbance. Shifting those flights to the other rotation is an attempt to reduce its delay, i.e. to decrease the value of the objective function. Interchanging parts of two rotations is possible only if it leads to a delay reduction. In order to interchange some parts of two rotations, it is necessary that these parts depart from the same airport within the time period from the planned departure time to the time caused by the maximal allowed delay of the considered flights. Let us mention that not only parts with the delayed flights can be interchanged, but also with delayed and no delayed flights if the total delay is reduced in this way. A special heuristic algorithm consists of the following steps.

Step 1 - Designing the basic feasible schedule: In order to create a new feasible daily operational flight schedule, all operating aircraft are considered, both aircraft that have landed at their arrival airports and aircraft that are in flight at the moment of the disturbance and which capacity is not smaller than the capacity of disturbed aircraft, on condition that the aircraft is from the same or from the first following category of aircraft. For each disturbed aircraft whose rotation does not contain priority flights, a new feasible schedule is designed as follows:

- If the delay is less than the maximal allowed, the basic feasible solution is designed by shifting the delayed flight by the delay time. The following flight departs either on time or after completing the previous flight.
- If the delay of flight is greater than allowed maximum, the basic feasible solution is designed so that the mini rotation containing the delayed flight is temporarily cancelled. If the other flights in that rotation do not have a delay greater than the allowed maximum, they are realized with or without delay. Mini rotations/simple segments of rotation with allowed delay are also temporarily cancelled if the constrain related to airport working hours is violated.
If the flight belongs to the simple segment of rotation, and delay is greater than allowed maximum, all flights until the end of that rotation are temporarily cancelled.

For each disturbed aircraft whose rotation contains a priority flight, a new feasible schedule is designed as follows:

- If delay of the priority flight is not greater than the maximum allowed one, then this flight is realized with this delay, while a following flight in the rotation depart either on time or it is shifted to start immediately after the completion of the previous flight.
- If the delay of the priority flight is greater than the maximal allowed one and there is a mini rotation(s) which precedes this flight, then this mini rotation(s) is temporarily cancelled, until the priority flight is serviced with the allowed delay.
- If the priority flight is part of a mini rotation assign the mini rotation of the considered priority flight to other aircraft from the same or from the first following category of aircraft.
- If it is not possible to find an aircraft which can realize the priority flight’s mini rotation, find an aircraft which can realize the priority flight using the VIA principle and which is from the same or from the first following category of aircraft as the originally assigned aircraft. When the priority flight is a part of simple segment of rotation, realize it in the same, abovementioned way.
- If none of the abovementioned procedure led to realization of the priority flight, cancel it permanently. If the considered aircraft has more than one priority flight in its rotation, repeat the previous procedure for each of them.

Now the complete basic feasible schedule is designed by repeating the entire Step 1 for each aircraft that is influenced by the disturbance, and the corresponding value of the objective function value is calculated. If there are no temporarily cancelled flights in the designed complete basic schedule, the heuristic algorithm goes to Step 3. Otherwise it goes to Step 2.

**Step 2 - Adding temporarily cancelled flights:** All temporarily cancelled flights are sorted in a list according to the following procedure: partition the set of all temporarily cancelled flights into the cancelled mini rotations and simple segments and sort them in the four groups. The groups are: (1) mini rotations that begin and end at the base airport of their rotations; (2) mini rotations that begin and end at some other airport; (3) simple segments that begin or end at the base airport of their rotation; (4) simple segments that neither begin nor end at the base airport of their rotations. After that lexicographically sort elements (mini rotations or single segments) of each group according to the decreasing total number of flights and the decreasing total number of passengers. Starting from the top of the list of the temporarily cancelled flights, the heuristic algorithm passes through all its elements and tries to add each of them to the basic schedule, determined in Step 1. The element (a mini rotation or a simple segment) should be added to the schedule within the time period between the planned departure time and the latest possible departure time (according to the maximal allowed delay or airport working hours) of the first flight in this element. The element is added to the rotation of an aircraft of the same or the first following category as the aircraft that was originally supposed to realize it in order to reduce total number of cancelled flights and additional costs. When the list of the temporarily cancelled flights is exhausted, the algorithm goes to Step 3 if there are the delayed flights in the current schedule. Otherwise, it stops.

**Step 3 - Rotation crossing:** All mini rotations and simple segments of the schedule, obtained by Step 2, which contain delayed flights, are sorted into the list of the delayed flights according to the total delay costs of passenger. The delay cost of passengers calculates as a product of number of passengers on flight, delay of flight obtained in the step 2 and average delay cost per minute for each delayed flight. Rotation crossing refers to removing parts of the rotation and adding them to some other rotation (the first part of Step 3) and interchanging parts of two rotations (the second part of Step 3) in order to reduce passenger’s delay costs. In this step the aircraft balance and the number of realized flights are not changed. As a result of Step 3 the final $n$ solutions list is offered to the dispatcher, sorted in a descending order of the objective function value. Then go to Step 4.

**Step 4:** The end of the algorithm.
4 Numerical example

As a result of the model application, at least one feasible solution is provided. Each solution is presented by a graphical representation of the daily schedule and table which includes planned and obtained data on the flight schedule (flight delay, flight cancellation, swapping registration or type of aircraft, etc.).

The algorithm is illustrated with reference to middle size European airline's timetable. The chosen values of penalties represent the airline policy and in numerical examples they are: penalty for priority flight cancellation $k(i)=200,000$ units per flight, penalty for non-priority flight cancellation $k_2(i)=100,000$ units per flight, penalty for aircraft balance disturbance $pen(l,k)=3,000$ units per aircraft, penalty for passenger delay per minute $p_k=1$ unit per passenger minute. In the considered example maximum allowed delay for domestic flight is 360 minutes and maximum allowed delay for international flight is 180 minutes. Within the given day, operations were executed by 29 aircraft (9 different aircraft types), assigned to 126 flights. Because there is no data about aircraft maintenance within the technical maintenance system for this day, it was assumed that all aircraft introduced into the realization of scheduled operations were available till the end of given day and the constraint related to the technical base is not considered. The costs of realization flight by appropriate aircraft type are calculated for each flight, and they are put, together with the other flight data, in the software data base. The solutions list offered to the dispatchers consists of a maximum of 10 solutions.

Example: Aircraft 11 is failed and it is being repaired from 14:00 till 20:00. This aircraft is planned to realize one priority flight 415 (A-C). Due to the limited space, graphical presentation of input and output data is omitted, while output results are given in table form. The software offered 6 solutions, but only first three solutions will be illustrated.

Solution 1: The disturbance directly affected priority flight’s mini rotation A-C-A (priority flight 415 and non-priority flight 416). In the first step of the algorithm, this mini rotation is assigned to aircraft 12, where both flights are realized on time, without delay. Flight 307 (A-P) will be delayed 123 minutes, because of servicing abovementioned mini rotation. There are no temporarily cancelled flights in the first step of the algorithm, so we move to the third step. In the first part of step 3, delayed flights are added in the rotation of the aircraft from the same or the first following category in order to reduce the value of objective function. In this case, the flight delayed in the step 1, is added to aircraft 13 where it is realized on time. All flights are realized on time, so there are no passenger’s delay costs, and value of objective function is equal to 0 (Table 1).

<table>
<thead>
<tr>
<th>Flight</th>
<th>415</th>
<th>416</th>
<th>307</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure airport</td>
<td>A</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>Arrival airport</td>
<td>C</td>
<td>A</td>
<td>P</td>
</tr>
<tr>
<td>Planned departure time</td>
<td>14:14</td>
<td>17:09</td>
<td>17:53</td>
</tr>
<tr>
<td>Real departure time</td>
<td>14:14</td>
<td>17:09</td>
<td>17:53</td>
</tr>
<tr>
<td>Delay (min)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A/C planned to realize the flight</td>
<td>11</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>A/C which actually realized the flight</td>
<td>12</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Number of passengers on flight</td>
<td>137</td>
<td>136</td>
<td>145</td>
</tr>
<tr>
<td>Value of objective function</td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Solution 2: The second solution is obtained after the first part of the step 3. In this solution also there are no delayed flights. In the first part of the step 3 (adding flights), flight 307 is assigned to aircraft 23 instead of 12. Because of aircraft type changing (flight 307) the cost of planned aircraft type swapping is appeared in the objective function. The aircraft balance is violated. So, the value of objective function is sum of the cost of planned aircraft type swapping and cost of aircraft balance disturbance.
**Solution 3**: The flight 307 (A-P) is realized, after step 1, with 123 minutes delay. In the second part of the step 3, this flight is realized on time by aircraft 14 (instead of 12), while 859 is performed with 91 minutes delay, by aircraft 12 (instead of 14). The objective function represents cost of passengers’ delay. Its value is 11.284 units and it is given in Table 2, as is data about flights influenced by disturbance.

Table 2. Data on flights influenced by disturbance, solution 3

<table>
<thead>
<tr>
<th></th>
<th>415</th>
<th>416</th>
<th>859</th>
<th>307</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure airport</td>
<td>A</td>
<td>C</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Arrival airport</td>
<td>C</td>
<td>A</td>
<td>TLV</td>
<td>P</td>
</tr>
<tr>
<td>Planned departure time</td>
<td>14:14</td>
<td>17:09</td>
<td>18:25</td>
<td>17:53</td>
</tr>
<tr>
<td>Real departure time</td>
<td>14:14</td>
<td>17:09</td>
<td>19:56</td>
<td>17:53</td>
</tr>
<tr>
<td>Delay (min)</td>
<td>0</td>
<td>0</td>
<td>91</td>
<td>0</td>
</tr>
<tr>
<td>A/C planned to realize the flight</td>
<td>11</td>
<td>11</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>A/C which actually realized the flight</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>Number of passengers on flight</td>
<td>137</td>
<td>136</td>
<td>124</td>
<td>145</td>
</tr>
</tbody>
</table>

Value of objective function $124.91 = 11.284$

All 6 offered solutions are given in Table 3. In the first and second solution, none of flights is delayed. Solutions 3-6 have one delayed flights. Total delay time goes from 0 to 138 minutes, while number of delayed passengers goes from 0 to 145. The value of objective function varies from 0 to 20.010 units. This DSS is installed on PC Pentium IV and it yields a set of solutions in less than 10 seconds.

Table 3. Data on all offered solutions

<table>
<thead>
<tr>
<th>Solution list</th>
<th>Number of delayed flights</th>
<th>Total delay (min)</th>
<th>Number of delayed passengers</th>
<th>Number of flights which are not realized according to flight plan</th>
<th>Value of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3.982,16</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>91</td>
<td>124</td>
<td>4</td>
<td>11.284,00</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>123</td>
<td>145</td>
<td>2</td>
<td>17.835,00</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>127</td>
<td>145</td>
<td>3</td>
<td>18.145,00</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>138</td>
<td>145</td>
<td>3</td>
<td>20.010,00</td>
</tr>
</tbody>
</table>

5. Conclusion and future directions

The airline schedule disturbance problem is a problem that airlines face daily. Because of the disturbance, airlines can have additional costs, passengers can be dissatisfied, and the reputation of an airline can be reduced. In order to minimize disturbance effects, the dispatchers in the AOC need to perceive the consequences of different solutions from all aspects, in real time. Decision support systems that can offer several feasible solutions based on airline policy in real time are needed in order to simplify the decision-making process.

This paper presents a mathematical formalization of the airline schedule disturbance problem and a special heuristic algorithm for designing a new daily operational flight schedule due to disturbances. This heuristic algorithm can be used in real time for generating several new daily schedules, which are sorted by decreasing value of the objective function. Any of these flight schedules could be accepted by dispatchers. The proposed model is supported by the corresponding software which is illustrated using a middle size European airline’s numerical example.

Based on the review of the developed DSS (i.e. mathematical formalization, the special heuristic algorithm, developed software with user-friendly interface) and numerical examples given in this paper, it can be concluded that: 1) The model yields as a result a list of feasible solutions in a real time, so the decision maker can choose and apply any of the solutions from...
the list considering, if necessary, criteria not included into the model. 2) Delaying, cancellation and resource (aircraft) substitution are suggested as the main actions for disturbance problem solving; using spare resource and ferry flights are not foreseen by the developed model. 3) The costs are not presented by real value, but penalties; the dispatcher can change these penalties according to experience, instantaneous traffic situation which has to be solved, or airline policy. New strategies can be defined by changing values of penalties (costs), therefore the DSS can accommodate different airline policies. 4) The models are usually made for specific airline needs; a great advantage of the model presented in this paper is the possibility of changing penalty values which can adapt the model for different airline needs. 5) The developed DSS is easy to use for less experienced dispatchers and a useful tool in dispatcher training. 6) The use of the developed decision support system in the AOC could make the dispatchers’ work simpler and faster, as well as foresee the effects of the applied solution.

The model presented in this paper gives solutions that are feasible from the aspect of aircraft availability, while crew availability is not considered despite of the close link between aircraft and crew. In further research these two resources could be merged, so that the obtained solution is feasible from the aspect of both aircraft and crew. This will facilitate the dispatchers’ work, because they could reallocate all resources by running single software. This would also additionally reduce the time needed for disturbance problem solving. Ferry flights can be introduced as well as spare aircraft use.

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References